To convert a set of three Euler angles, ϕ_1, ϕ_2, ϕ_3 (1, 2, 3 are the first, second, third Euler rotations, not the axes of rotation) to the equivalent quaternion:

Note: You must know the Euler rotation axis sequence, i.e
123, 321, 213, 121, etc.

1) form three quaternions from the three Euler angles:
   a. for a "1" rotation axis, the quaternion is
      $\sin(\phi/2)$ 0.0 0.0 $\cos(\phi/2)$
   b. for a "2" rotation axis, the quaternion is
      0.0 $\sin(\phi/2)$ 0.0 $\cos(\phi/2)$
   c. for a "3" rotation axis, the quaternion is
      0.0 0.0 $\sin(\phi/2)$ $\cos(\phi/2)$

2) multiply the three quaternions in the correct order.

for example,

given:

rotation order 312
$\phi_1 = 30$ deg
$\phi_2 = 60$ deg
$\phi_3 = 45$ deg

Q_1 = 0.0 0.0 sin(30/2) cos(30/2) = 0.0 0.0 0.258819045 0.965925826
Q_2 = sin(60/2) 0.0 0.0 cos(60/2) = .5 0.0 0.0 0.866025404
Q_3 = 0.0 sin(45/2) 0.0 cos(45/2) = 0.382683432 0.0 0.0 0.923879533

$Q_f = Q_1\ Q_2\ Q_3$
= 0.360423406 0.43967974 0.391903837 0.723317411